# Shark Fin Option Pricing - Based on Monte-Carlo Simulatio

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**Abstract:** Shark fin option has received a good response since its launch. It can function as a combination of options on bonds due to its guaranteed income. There may be a satisfactory gain when the expected fluctuations are not particularly dramatic. Until now, many financial institutions are selling the product. And the underlying assets have also expanded from the initial stock index to a broader range, such as the exchange rate. But option pricing has always been a hot topic. Shark fin option is essentially an upward knock-out option. The pricing of European shark fin option needs to compare the ending strike price and market price, but the fluctuation in market price is uncertain. Therefore, this paper starts with a financial product of shark fin option and prices it through Monte-Carlo Simulation. Monte-Carlo Simulation is essentially a continuous binary tree, whose core idea is to take the average as the final result through numerous simulations.

## 1. Introduction

Shark fin option, also known as knock-out option, belongs to a kind of barrier option. The option contract sets the price range for the underlying asset in advance. If the underlying asset price falls in the range within the time frame agreed in the contract, the option is an ordinary call or put option; Once the underlying asset price jumps out of the range, the option will be automatically knocked out[1].

Depending on the number of barrier prices, shark fin options can be divided into single shark options and double shark options, and the single shark option value sets a barrier price[2]. Taking a call single shark fin option as an example, if a knock-out price is set, the option will be knocked out automatically when the underlying price is higher than the knock-out price; Double shark fin options are two barrier options. As long as the underlying asset price is not between the two barrier prices, the option will be automatically knocked out. Shark options are also classified based on the knock-out price and the relative size of underlying assets at the initial moment: If the knock-out price is higher than the initial price, call it an upward knock-out option. It is a downward knock-out option on the contrary.

The upward knock-out option sets the barrier level B (B>) above the initial price S0 of the subject matter on the basis of the ordinary European option. Options automatically disappear when the asset price exceeds the barrier price. Depending on investors' expectations on the future trend of the subject matter, upward knock-out options can be divided into upward knock-out call option and upward knock-out put option.

Investors buy an upward call option with an exercise price of k, and a barrier price of B. If the price of the underlying asset is higher than B during the contract period, the option will automatically disappear and never be exercised. At this time, the option price is 0 and the designed option will be meaningless. The option is only possible to exercise when B<K. Assuming that the option has a premium of c, the underlying asset price when the contract expires is ST. If the price of the underlying asset never exceeds the barrier price B and the expiration price is lower than the exercise price K during the contract period, the long party will give up the exercise and the return is - C, that is, the premium of the option is lost; If the price of the underlying asset never exceeds the barrier price B during the contract period and the expiration price is higher than the exercise price

K, the yield to maturity is ST-K-c; If the price of the underlying asset exceeds the barrier price during the contract period, the option will disappear automatically, and the loss of the investor is also the option premium.

Table 1Income of upward knock-out option

| | Expiration price of underlying assets | Long earnings of call option |
|---|---|---|
| The underlying asset price does not exceed the barrier price during the contract period | $S_T <= K$ | -c |
| maxS(t)<=B(0<t<T) | $S_T > K$ | $S_T$-$K-c$ |
| The underlying asset price exceeds the barrier price during the contract period | $S_T <= K$ | -c |
| maxS(t)>B(0<t<T) | $S_T > K$ | -c |

The income structure diagram is as follows:

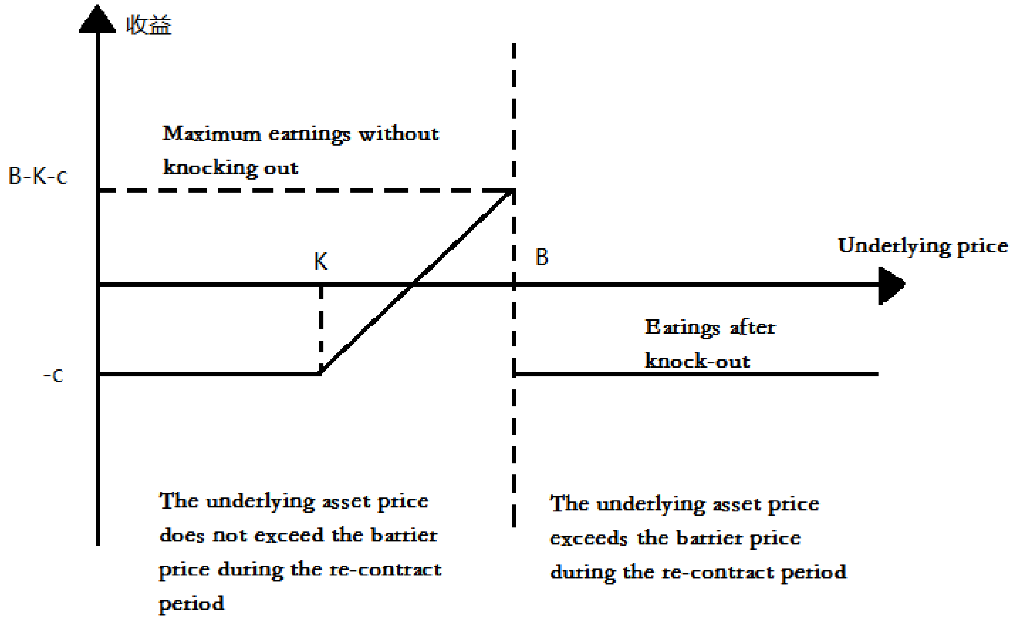


Figure 1Income structure diagram of upward knock-out option

## 2. Option pricing

Taking the up strike call option as an example, the explicit expression of shark fin option pricing is introduced. Assuming the premium of upward knock-out call option with an exercise price K and a barrier price B is $c_0$, the initial price of underlying assets is $S_0$. Assuming that the price change of the underlying asset is Brownian motion, using the nature of Brownian motion and the theory of random distribution, the pricing formula of the upward knock-out call option can be deduced as follows[2]:

$$c_0 = S_0[N(\delta,(T,\frac{S_0}{K}) - N(\delta,(T,\frac{S_0}{B}))]$$

$$-e^{-rT}[N(T,\frac{S_0}{K}) - N(\delta,(T,\frac{S_0}{B}))]$$

$$-B(\frac{S_0}{B})^{-\frac{2r}{\sigma^2}}[N(T,\frac{B^2}{KS_0}) - N(\delta,(T,\frac{B}{S_0}))]$$

$$+e^{-rT}(\frac{S_0}{B})^{-\frac{2r}{\sigma^2}+1}[N(T,\frac{B^2}{KS_0}) - N(\delta,(T,\frac{B}{S_0}))]$$

Where, N (*) represents the accumulation of the standard normal distribution function;

$$\delta(\tau, s) = \frac{1}{\sigma\sqrt{\tau}}[\ln(S) + (r \pm \frac{1}{2}\sigma^2)\tau]$$

T stands for the remaining time of option, r for risk-free interest rate and σ for the volatility of underlying assets.

So far, the above shark fin options are all standard forms, that is, the income is zero after the option is knocked out; In practice, considering the needs of investors, many shark fin options are designed to take the form with knock-out income. When the asset price reaches the barrier level, the buyer obtains a compensation income upon the option expiration, which is called knock-out income. When the barrier option has a knock-out income, the applicable probability method discounts the knock-out income according to the density function of the first touch barrier, and the knock-out income can be obtained[3]. Taking the downward knock-out call option as an example, the option knock-out probability can be known, i.e.:

$$\tilde{P}\{\tilde{M}(T) >= b\} = 1 - N(\frac{b - \alpha T}{\sqrt{T}}) + e^{2\alpha b} N(\frac{-b - \alpha T}{\sqrt{T}})$$

In which,

$$\alpha = \frac{1}{\sigma}(r - \frac{1}{2}\sigma^2), b = \frac{1}{\sigma}\log\frac{B}{S(0)}$$

So the price of the knock-out options is

$$c_{Rchate} = e^{-rT} R \times \tilde{P}\{\tilde{M}(T) >= b\}$$

R standards for the knock-out income. This income structure is very common in real transactions. If a shark fin option has a knock-out income, it will need to be split into two parts for pricing: One is not to consider the knock-out income; The second is to knock out part of the income. The complete option price is expressed as CALL, so:

$$c_{ALl} = c_0 + c_{Rchate}$$

## 3. Monte-Carlo Simulation

Monte-Carlo is a numerical method that simulates the random movement of the underlying asset price and gets only the expected value of the pricing product, and discounts this expected value to the present point to estimate the derivative price. It is an idea to get a random cash flow through multiple simulations, discount it through the risk-free interest rate, and take the present value of the cash flow obtained multiple times to average as the option price. Its essence is also a pricing method under the risk-neutral principle[3].

The advantage of Monte Carlo is that it can price various complex derivatives with a straightforward idea. However, when using this method to simulate the path of the underlying asset price and price the shark fin option, there is a problem that must be paid attention to: For any path to determine whether the barrier option is knocked out in this case, you need to compare the maximum (or minimum) of the underlying asset price to the barrier price[4]. Therefore, when conducting the Monte-Carlo simulation, we not only need to know the price of the underlying asset at each time node, but also need to determine whether the underlying asset is knocked out in this range. However, in the ordinary Monte-Carlo simulation process, only the price of nodes can only be simulated, and the maximum underlying price in this range cannot be known. Therefore, it is impossible to accurately judge whether the target has exceeded the barrier price in this range, resulting in inaccurate pricing.

In order to improve the accuracy of pricing, an improved Monte-Carlo method is described here, taking the knock-out call option as an example: For each path (the number of time nodes is n), the underlying asset price follows Brownian motion[5].

$$S_T = S_0 \exp[(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t]$$

Where $S_T$ is the price of the underlying asset at time t, $S_0$ is the initial price of the underlying asset, μ is the rate of return on the underlying asset, σ is the volatility or standard deviation of the underlying asset price, Wt is a Wiener process, also known as Brownian motion. In short, Wt obeys the normal distribution with a mean value of 0 and a variance of $t\sigma^2$. It can also be understood as the sum of t standard normal distributions.

The probability Pk that the asset price is not less than the barrier price B in the time interval $[_k, t_k+t]$ can be calculated, and the uniformly distributed random number $u_k$ on 0-1 can be generated at the same time, including:

$$P_{k+t} = P[\max_{t \in [t_k, t_{k+t}]} S_t >= B] = \exp(-2\frac{(B-S_k)(B-S_{k+1})}{\sigma^2 S_k^2 (t_{k+t} - t_k)})$$

## 4. Financial Products of Shark Fin Options

### 4.1. The role of enterprise market expansion

Take the bullish shark fin RMB financial products linked to the CSI300 index issued by the Bank of China as an example to show the characteristics of such financial products in detail.

| Basic Attributes | Terms |
|---|---|
| Currency | RMB |
| Income type | Non breakeven floating type |
| Investment varieties | Stocks, funds, bonds, interest rates, others |
| Linked object | CSI300 Index |
| Anticipated annual rate of return | 3.2%~8.8% |
| Interest payment method | Expired payment |
| Start date of income | 2018-08-09 |
| Maturity date of income | 2018-11-07 |
| Duration | 90 days |
| Entrusted initial fund | RMB 50,000 |
| Description of rate of return | (1) If for the single-phase products, the ratio of the closing price of the linked subject matter in the observation period to the closing price of the linked subject matter on the starting date of the current product investment has been greater than the barrier price (the initial price is about 107%), the expected annualized return of the product is equal to the expected knock-out rate of return of 5%;<br>(2) closing price of the linked subject matter in the observation period to the closing price of the linked subject matter on the starting date of the current product investment has never been greater than the barrier price, the calculation plan of the expected annualized rate of return of the product is as follows: If for the single-phase products, the ratio of the closing price of the linked subject matter in the observation period to the closing price of the linked subject matter on the starting date of the current product investment is less than the exercise price, the expected annualized rate of return is equal to the expected minimum rate of return of 3.2%; If for the single-phase |

| | |
|---|---|
| | products, the ratio of the closing price of the linked subject matter in the observation period to the closing price of the linked subject matter on the starting date of the current product investment is larger than or equal to the exercise price, the expected annualized rate of return is equal to the expected minimum rate of return of 3.2%+80%* (The closing price of the linked target on the observation date of the current product / the closing price of the linked target on the starting date of the current product investment-1)<br>Investor's interest income = financial management funds * annualized rate of return to maturity * actual financial management days / 365 |
| Description of fees | (1) Custody fee: 0.05%/year, collected by the custodian on a daily basis;<br>(2) Sales management fee: 0.50%/year |
| Product introduction | Financial products (I) Investment scope 1. Fixed income Treasury bonds, financial bonds, central bank bills, high-grade credit bonds, bond funds, asset-backed securities and other fixed-income products; 2. Monetary market: Inter-bank deposits, interbank borrowing, bond repurchase, money funds and other money market assets; 3. Trust plan and other assets or portfolio meeting institutional requirements; 4. Financial derivatives. (II) Investment proportion: the investment proportion of fixed income and money market assets is 10% - 100%, and the investment proportion of the portfolio and financial derivatives in the trust plan is no more than 90%. |

The exercise price in the above table is equal to the initial price of the underlying asset. The barrier price is 107% of the initial price, with a knock-out rate of return of 5% and the lowest rate of return of 3.2%. Under normal circumstances, the financial product can obtain at least 3.2% income. The additional income of investors is determined by the performance and the closing price of CSI300 index on the maturity date. Therefore, the financial product can be composed of the following two parts[5]:

1) Fixed income:

Regardless of the maturity performance of the linked object, investors can obtain an annualized return at 3.2% of the principal. This part can be seen as investors buying a zero coupon bond with an annualized rate of return of 3.2% at the beginning.

2) 0.8 upward knock-out call option:

This portion of earnings is linked to the performance of the underlying assets. According to the rate of return in the product treaty, the closing price of CSI300 is determined to be 3,397.53 points on the starting date of the income of financial products (August 9, 2018); The maturity date of the financial products is November 7, 2018. If the closing price of CSI300 index was higher than 3635.36 points during this period, the investors of the financial product can obtain the upward knock-out call option with 0.8 knock-out rate of return of 2.25% on the basis of 3.2% fixed income, that is, the annualized rate of return is 5% (5%=3.2%+0.8*2.25%); If the closing price of the underlying contract is never higher than 3635.36 points within the product term, and is lower than the exercise price of 3397.53 points upon expiration, the investor can only obtain a fixed rate of return of 3.2%; If the target has never exceeded the barrier price and the maturity price is higher than 3397.53 points, the investor's rate of return is 3.2% + 80% * the performance of CSI300 index.

The product has 0.8 shark fin options with knock-out income (knock-out rate of return is 2.25%), which is different from the barrier option pricing without knock-out income under the standard form. It is worth noting that during the contract period, the product needs to compare the closing price of the underlying asset with the barrier price every day to judge whether the option is knocked out, while the standard shark fin option is knocked out as long as the price is higher than the barrier price at any time, which is equivalent to the relaxation of the knock-out conditions of the option. Since this report focuses on the analysis of shark fin option products, the option approximation embedded in this product is considered as a standard shark fin option here.

The income structure diagram is as follows:

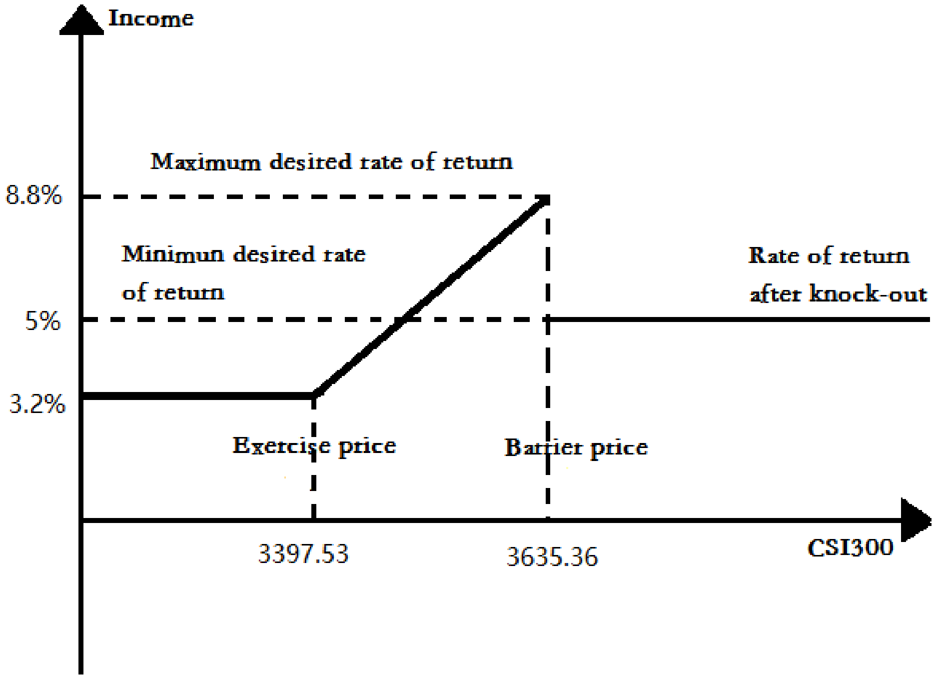


Figure 2 Income structure chart of shark fin option linked to CSI300 index

After determining the income structure of this product, this paper splits the product into zero coupon bonds and bullish shark fin options with 0.8 knock-out returns, and discounts and discusses future cash flow respectively. Select the average one-year treasury bond rate of return during the contract period of 2.89% as the risk-free interest rate. If the investor's principal is RMB 50,000, then an annualized income of 3.2% can be obtained at maturity. The fixed income portion can be considered as zero coupon bonds with their value discounted as:

$$\frac{50000+50000\times 3.2\%\times 90/365}{(1+2.89\%)}=50041.74$$

For a bullish shark fin option (an upward knock-out call option), the exercise price of the option is the same as the initial price of the underlying asset, K=S0 and knock-out price B=3635.36. The final return of the option is determined based on whether the maximum price of the underlying asset during the contract exceeds the barrier price. Assume no premium

Table 2Income table of shark fin option linked to CSI300 index

| | Expiration price of underlying assets | Long earnings of call option |
|---|---|---|
| The underlying asset price does not exceed the barrier price during the contract period | $S_T <= 3397.53$ | 0 |
| maxS(t)<=3635.36(0<t<T) | $S_T > 3397.53$ | $0.8*(S_T - K)/S_0$ |
| The underlying asset price exceeds the barrier price during the contract period | $S_T <= 3397.53$ | $0.8*2.25\%$ |
| maxS(t)>B(0<t<T) | $S_T > 3397.53$ | $0.8*2.25\%$ |

## 4.2. Simulation results

Monte-Carlo model needs to know the volatility of CSI300 index. In this paper, the volatility of the continuous daily rate of return is selected to calculate the volatility of annual rate of return.

$$\sigma_Y = \sqrt{252}\sigma_D$$

In which, $\sigma_Y$ stands for the volatility of annual rate of return; $\sigma_D$ for the volatility of daily rate of return; 252 means the number of trading days a year.

Calculate the daily rate of return with 252 days as the cycle through Python for the closing price, and then multiply it by the square of the date to obtain the volatility of the annual rate of return. Take the average value as 0.024792609243941952, and leave two decimals as 0.025.

The closing price, continuous day rate of return and volatility of continuous day remuneration rate are as follows:

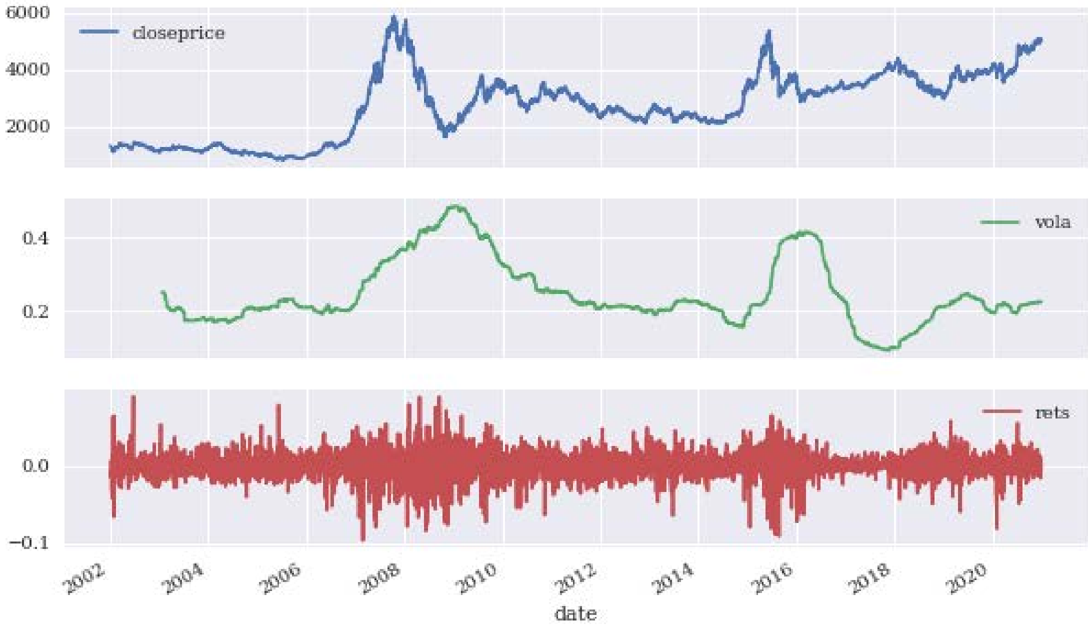


Figure 3 Simulated result

The option price can be calculated using the formula in 2.1 or the Monte-Carlo simulation by python. Here we show the results obtained by the ordinary Monte-Carlo method and the improved method, respectively. It is not difficult to see that the product is issued at a discount.

The product has already expired. We know that the closing price of CSI300 index on November 7, 2018 was 3,221.91 points, lower than the initial price of the underlying asset. The maximum closing price of the product during the contract period was 3,438.87 points on September 28, which did not touch the agreed barrier price. According to the product treaty, the annualized rate of return of investors when the option expires is 3.2%, and all the ending funds come from fixed income investment.

Table 3Comparation of the Expression calculation and Monte-Carlo Simulation

| Item | Expression calculation | Monte-Carlo Simulation |
|---|---|---|
| Principal | 50000 | 50000 |
| Fixed-income value | 50041.74 | 50041.74 |
| Bullish shark fin option value | 144.16 | 132.83 |
| Bank charges | -67.81 | -67.81 |
| Product discount value | 50118.89 | 50106.76 |

## 5. Conclusions

With the development of financial market, investors are no longer limited to simple financial products, but more pursue debt structured products, which not only requires financial institutions and financial managers to pay more attention to innovation and investor demand preference.However, innovative financial products often have certain pricing difficulties. First, there are no related products in the market and there is a lack of experience for reference. Second, it is impossible to make reasonable predictions of all the situations. For example, citic Pacific's Australian project did not anticipate the financial crisis and did not have a corresponding risk defense mechanism, which led to pricing errors and large-scale losses of the project. The third is to determine a reasonable pricing method. There are many options pricing methods, but each method has certain limitations.To sum up, Monte Carlo simulation has good simulation effect on European barrier options, and will have better simulation effect through future improvement.

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